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# Estimating the Vibration Level of an L-Shaped Beam Using Power Flow Techniques

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The response of one component of an L-shaped beam, with point force excitation on the other component, is estimated using the power flow method. The transmitted power from the source component to the receiver component is expressed in terms of the transfer and input mobilities at the excitation point and the joint. The response is estimated both in narrow frequency bands, using the exact geometry of the beams, and as a frequency averaged response using infinite beam models. The results using this power flow technique are compared to the results obtained using finite element analysis (FEA) of the L-shaped beam for the low frequency response and to results obtained using statistical energy analysis (SEA) for the high frequencies. The agreement between the FEA results and the power flow method results at low frequencies is very good. SEA results are in terms of frequency averaged levels and these are in perfect agreement with the results obtained using the infinite beam models in the power flow method. The narrow frequency band results from the power flow method also converge to the SEA results at high frequencies. The advantage of the power flow method is that, detail of the response can be retained while reducing computation time, which will allow the narrow frequency band analysis of the response to be extended to higher frequencies.

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ESTIMATING THE VIBRATION LEVEL OF AN L-SHAPED
BEAM USING POWER FLOW TECHNIQUES

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## INTRODUCTION

The response of a finite structure to harmonic excitation
may be analyzed by different methods depending on the frequency
range of interest. Finite element analysis (FEA) gives good
results at low frequencies but at higher frequencies, the number
of elements required to obtain reasonable results becomes
intractable.  Statistical energy analysis (SEA), on the other
hand, is useful at high frequencies where high modal density
gives a fairly smooth response with frequency.  The power flow
method, however, may be used at low frequencies, where resonances
are important and at high frequencies, where the modal density is
high.  In addition, the power flow method retains detail of the
response while reducing computation time, which will allow the
narrow frequency band analysis of the response to be extended to
higher frequencies.
To demonstrate the power flow approach, the response of one
component of an L-shaped beam, subjected to point force
excitation on the other component, is estimated using the power
flow method, and compared to the response predicted by the FEA
and SEA techniques.  The transmitted power from the source
component to the receiver component is expressed in terms of the
transfer and input mobilities at the excitation point and the
joint. The response is estimated both in narrow frequency bands,
using the exact geometry of the beams, and as a frequency
averaged response using infinite beam models.  The results using
this power flow technique are compared to the results obtained
using FEA of the L-shaped beam for the low frequency response and
to results obtained using SEA for the high frequencies.  The
results from these three methods are described in the following
sections.

## POWER FLOW METHOD

The power transmitted to the receiver beam is given by [1]

$$P_{trans} = \frac{1}{2} \, |F(f)|^2 \, \left| \frac{M_{F-\theta}}{M_2 + M_3} \right|^2 \, \text{Real } \{M_3\} \qquad 1.$$

where $M_2$ and $M_3$ are the point mobilities at 2 and 3 respectively, (figure 1), and these point mobility are defined by the ratio of angular velocity to applied torque.
$M_{F-\theta}$ is the transfer mobility defined by the ratio of angular velocity at point 2 to the excitation force ($F(f)$) when source structure is isolated from receiver structure.

The power input to the source beam is given by [1]

$$P_{input} = \frac{1}{2} \, |F(f)|^2 \, \text{Real } \left\{ M_1 + \frac{M_{M-v} \, M_{F-\theta}}{M_2 + M_3} \right\} \qquad 2.$$

where $M_1$ is the input point mobility at 1 defined by the ratio of transverse velocity to excitation force ($F(f)$). $M_{M-v}$ is the transfer mobility defined by the ratio of the transverse velocity at the free end to the applied moment at the joint (M). $M_{M-v}$ has the same expression as $M_{F-\theta}$ due to reciprocity.

The point and transfer mobilities are evaluated by solving the differential equations (D.E.) of motion, which for a beam are of the form [2]:

$$\alpha^4 \, \frac{\partial^4 \xi}{\partial x^4} + \frac{\partial^2 \xi}{\partial t^2} = 0 \qquad 3.$$

Where $\alpha^4 = EI/m$ \qquad\qquad 4.

E is the elastic modulus
I the second moment of area about the neutral axis
m is the mass per unit length of the beams
and $\xi$ is the transverse displacement
This D.E. has the general solution
$\xi = A \cosh kx + B \sinh kx + C \cos kx + D \sin kx$
The boundary conditions are used in determining the constants A,B,C,D of the general solution.

Assuming that the beams are pinned at the joint, then at this point the displacement $\xi = 0$, and the bending moment $\partial 2\xi/\partial x2 = 0$. Damping is introduced by using a complex modulus

$$E = E \, (1+j\eta)) \qquad 5.$$

where $\eta$ is the loss factor.

## FINITE ELEMENT ANALYSIS

Two identical steel beams, rigidly connected in a right angle configuration, are modelled using the MARC finite element program. [3] The joint is externally pinned, allowing only in-plane rotation. The natural frequencies and mode shapes are evaluated up to a frequency of 1000 Hz. The assembly is subsequently subjected to harmonic excitation, and the response computed.

The beams are modelled using nineteen, 2-node Euler-Bernoulli beam elements with linear elastic material properties. The elastic modulus is a real constant. Structural damping is incorporated through the use of the loss factor ($\eta=0.01$). The beams are held at right angles by a brace near the corner joint. The beams have the material properties of steel, while the brace is significantly stiffer and lighter than steel in order to limit the effect of the brace on the natural frequencies of the structure. Each node has three coordinates in space, and each beam has ten nodes, equally spaced except in the vicinity of the joint and at the beam ends. Motion is restricted to transverse vibration so that only simple bending modes are considered.

A harmonic force, with frequencies ranging from 1 to 1000 Hz, is applied at the end of one of the beams. The frequency interval is 1 Hz in the frequency region 1 to 9 Hz, and 10 Hz between the frequencies of 10 to 1000 Hz. At each frequency the displacement amplitudes of the nodes in the receiver beam are used to compute the spatial average response which is related to the power dissipated. Since the receiver substructure is not connected to any other substructures, the dissipated power must equal the transmitted power.

## STATISTICAL ENERGY ANALYSIS

The L-shaped beam may be considered as a system of two coupled substructures. Writing the power flow balance equations [4]:

$$P_{in_i} = P_{diss_i} + P_{trans_{ij}} \qquad 6.$$

where     $i = 1, 2; \; j = 1, 2; \; i \neq j$

The power flow, and the power dissipated are both functions of the modal energies of the substructures:

$$P_{diss_i} = \omega \eta_i \langle E_i \rangle \qquad 7.$$

where     $i = 1, 2$

and for linear coupling between the two substructures:

$$P_{trans_{12}} = \omega \eta_{12} (\langle E_1 \rangle - \langle E_2 \rangle) \qquad 8.$$

where $\eta_{12}$ is the coupling loss factor of the substructures and $\eta_1$ and $\eta_2$ are the internal loss factor of substructures 1 and 2 respectively.

If only one substructure is subjected to external excitation then $P_{in_2} = 0$. The ratio of the power output to the power input is thus given by

$$\frac{P_{trans_{12}}}{P_{in_1}} = \frac{\eta_2 \, \omega \, \langle E_2 \rangle}{P_{in_1}} = \frac{\eta_2}{\left[ \frac{(\eta_1 + \eta_{12})(\eta_2 + \eta_{21})}{\eta_{21}} - \eta_{12} \right]} \qquad 9.$$

The transmitted power is in this case taken to be equal to the dissipated power. The coupling loss factors can be related to the junction transmission coefficient by the expression [5].

$$\eta_{12} = \frac{2}{\pi} \left( \frac{L_{12}}{k_1 A_1} \right) \tau_{12} \qquad 10.$$

Where $L_{12}$ is the length of the joint and $k_1$ and $A_1$ are the wave number and area of the source structure respectively. $\tau_{12}$ is the transmission coefficient defined by the ratio of transmitted to incident energy. $\tau_{12}$ can be calculated using a wave representation. Then

$$\eta_{12} = \frac{4}{\pi} \frac{L_{12}}{A_1} \left[ \frac{1}{\frac{D_2 \, k_2}{D_1 \, k_1} + 1} \right]^2 \cdot \frac{D_2 \, k_2}{D_1 \, k_1^2} \qquad 11.$$

and $\quad \eta_{12} = (\eta_2/\eta_1) \, \eta_{21} \qquad 12.$

Substituting equations (11) and (12) into (9) an expression for the ratio of transmitted to input power is obtained.

## RESULTS AND CONCLUSIONS

The results from the above methods of analysis are shown in figures 2 and 3. In figure 2, the results from the FEA are compared with those from the power flow method. Using the FEA method and keeping the analysis within reasonable bounds it was not possible to evaluate the response of the structure at frequency spacings as close as those used in the power flow method. Keeping in mind that the frequency resolution used was 1 Hz at low frequencies and 10 Hz between 10 and 1000 Hz, the agreement in the results is quite good. At high frequencies the disagreement is attributed to the limited number of elements and the introduction of the brace in the FE model. Below the first natural frequency the disagreement is not an error of the power flow method, it is caused by the definition of the loss factor. Using the power flow method the transmitted power is calculated from the product of the torque and angular displacement for the

receiver beam while using the FEA method the transmitted power is said to be equal to the dissipated power. This is only true above the first natural frequency. This result was checked using a closed form solution to the global L-shaped beam structure and the same discrepancy is obtained below the first natural frequency between the transmitted and dissipated power.

Figure 3 gives the results of the ratio of transmitted to input power for the SEA compared with that for the power flow method. As expected the details of the analysis are lost when the SEA method is used. Also, the SEA method can result in significant under or over estimates for the power transmitted to a particular substructure. The variations in the results will increase if the structure has a low loss factor. In the analysis, a loss factor of 0.01 was assumed. The power flow method curve shown in figure 3 asymptotically approaches the SEA result as the frequency increases.

The advantages of using the power flow method over other methods can be deduced from these results and from consideration of the computational efficiency. If the FEA were to be carried out with the same frequency resolution as was used in the power flow analysis, the latter method would be vastly more efficient computationally, than the FEA method. The SEA method is more efficient computationally; however it is unreliable as low frequencies since the fluctuation from the mean can be significant.

In conclusion, the power flow method is shown here to be a very powerful analysis technique. Although only demonstrated for two simple substructures, it is a simple matter to extend to multiple substructures, with multiple joints. The results produced have clearly demonstrated the usefulness of the power flow method at middle frequencies where the SEA methods can be unreliable and FEA methods become intractable.

## ACKNOWLEDGEMENT

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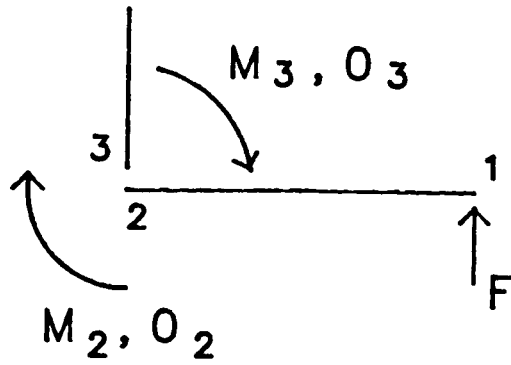5) R.H. Lyon Statistical Energy Analysis of Dynamic Systems, (MIT Press, 1975).
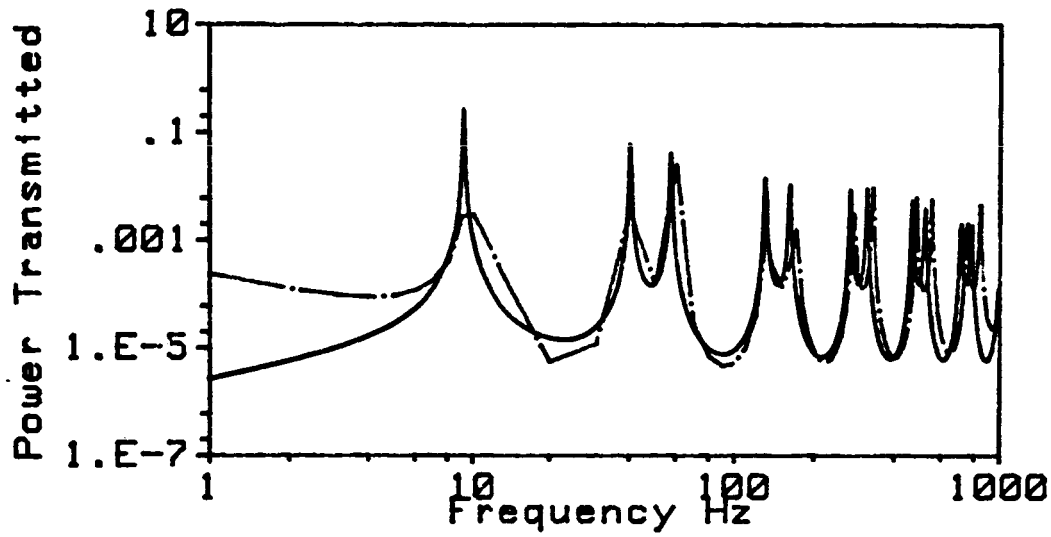
Figure 1.

Power Flow Model
for L-Shaped
Beam Structure



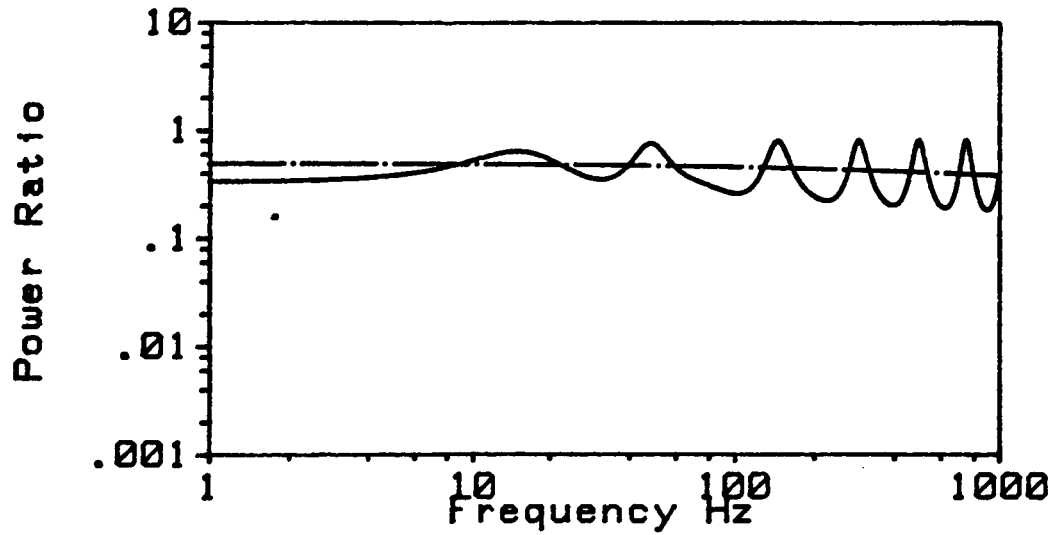Figure 2.  Comparison of Power Flow results (＿＿＿) to
FEA (---) results



Figure 3.  Comparison of Power Flow results (＿＿＿) to
SEA (---) results.